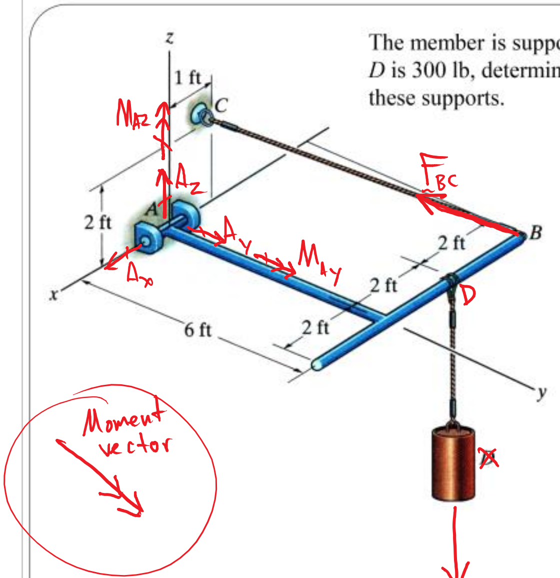


The member is supported by a pin at A and cable BC. If the load at D is 300 lb, determine the x, y, z components of the reactions at these supports.

$\underset{\sim}{W}_D = -300\hat{k}$ lbs

Reactions

A_x, A_y, A_z

M_{Ay}, M_{Az}

1 add'l unknown

$\underset{\sim}{F}_{BC} = F_{BC} \cdot \hat{u}_{BC}$

magnitude is unknown

Position vectors

$\underset{\sim}{r}_{AB} = (-4\hat{i} + 6\hat{j})$ ft

$\underset{\sim}{r}_{AD} = (-2\hat{i} + 6\hat{j})$ ft

$\underset{\sim}{r}_{BC} = (3\hat{i} - 6\hat{j} + 2\hat{k})$ ft

$$\hat{u}_{BC} = \frac{(3\hat{i} - 6\hat{j} + 2\hat{k})}{\sqrt{\underbrace{9 + 36 + 4}_{49}}}$$

$$= \frac{3}{7}\hat{i} - \frac{6}{7}\hat{j} + \frac{2}{7}\hat{k}$$

Force vectors

$\underset{\sim}{F}_{BC} = F_{BC}\left(\frac{3}{7}\hat{i} - \frac{6}{7}\hat{j} + \frac{2}{7}\hat{k}\right)$

$\underset{\sim}{W}_D = -300\hat{k}$ lbs

$\underset{\sim}{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$

$\underset{\sim}{M}_A = 0 \cdot \hat{i} + M_{Ay}\hat{j} + M_{Az}\hat{k}$

6 eqns.

$$\left.\begin{matrix} \Sigma F_x & \Sigma M_x \\ \Sigma F_y & \Sigma M_y \\ \Sigma F_z & \Sigma M_z \end{matrix}\right\} = 0$$

$\Sigma F_x = A_x + \frac{3}{7} \cdot F_{BC} = 0 \Rightarrow A_x = -\frac{3}{7}F_{BC}$

$\Sigma F_y = A_y - \frac{6}{7}F_{BC} = 0 \Rightarrow A_y = \frac{6}{7}F_{BC}$

$\Sigma F_z = A_z + \frac{2}{7}F_{BC} - (300\text{lbs}) = 0$

$\Rightarrow A_z = (300\text{lbs}) - \frac{2}{7}F_{BC}$

$$\sum \underset{\sim}{M}_A = M_{Ay}\cdot\hat{j} + M_{Az}\cdot\hat{k} + \underbrace{\underset{\sim}{r}_{AD} \times \underset{\sim}{W}_D} + \underbrace{\underset{\sim}{r}_{AB} \times \underset{\sim}{F}_{BC}} = \underset{\sim}{0}$$

$$\underset{\sim}{r}_{AD} \times \underset{\sim}{W}_D = (-2\hat{i} + 6\hat{j})\,ft \times (-300\hat{k})\,lbs = (-1800\hat{i} - 600\hat{j})\,lb\text{-}ft$$

$$\underset{\sim}{r}_{AB} \times \underset{\sim}{F}_{BC} = (-4\hat{i} + 6\hat{j})\,ft \times F_{BC}\cdot\left(\frac{3}{7}\hat{i} - \frac{6}{7}\hat{j} + \frac{2}{7}\hat{k}\right)$$

$$= F_{BC}\cdot\left(\frac{12}{7}ft\cdot\hat{i} + \frac{8}{7}ft\cdot\hat{j} + \frac{6}{7}ft\,\hat{k}\right)$$

In $\hat{i}$: $\quad 0 = -1800\,lb\cdot ft + F_{BC}\cdot\frac{12}{7}ft$

$$\Rightarrow \boxed{F_{BC} = 3150\,lbs}$$

In $\hat{j}$: $\quad 0 = M_{Ay} - (600\,lb\cdot ft) + F_{BC}\cdot\left(\frac{8}{7}ft\right)$

$$\Rightarrow \boxed{M_{Ay}} = 600\,lb\text{-}ft - (3150\,lbs)\left(\frac{8}{7}ft\right) = \boxed{-3000\,lb\cdot ft}$$

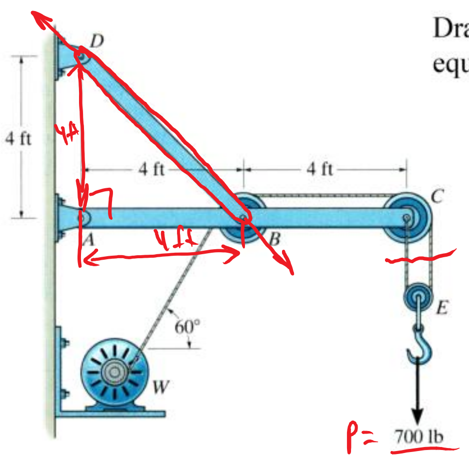
In $\hat{k}$: $\quad 0 = M_{Az} + F_{BC}\cdot\left(\frac{6}{7}ft\right)$

$$\Rightarrow \boxed{M_{Az} = -2700\,lb\cdot ft}$$

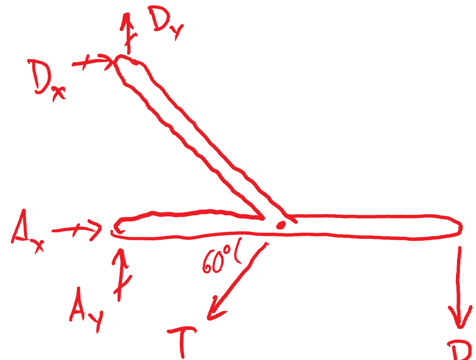
$$\boxed{A_x = -\frac{3}{7}(3150\,lbs) = -1350\,lbs}$$

$$\boxed{A_y = \frac{6}{7}(3150\,lbs) = 2700\,lbs}$$

$$\boxed{A_z = (300\,lbs) - \frac{2}{7}(3150\,lbs) = -600\,lbs}$$



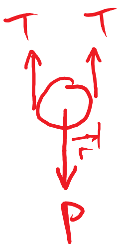
Draw a free-body diagram of the entire machine and set it into equilibrium. Neglect pulley sizes. Let P denote the 700 lb load.



Unknowns

A_x, A_y, D_x, D_y, T

FBD of E



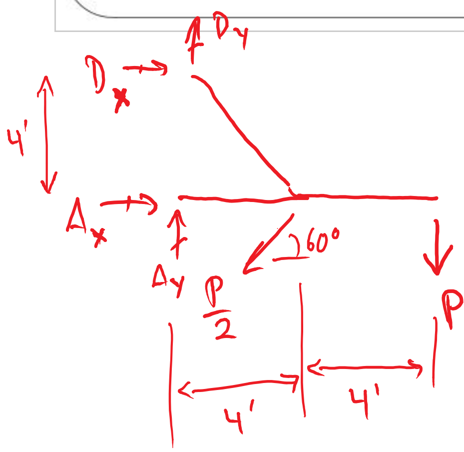
Why tension T on both sides?

$$\Sigma M_E = 0 \cdot P + r \cdot T_R - r \cdot T_L = 0$$

$$r(T_R - T_L) = 0$$

$$\Rightarrow T_R = T_L = T$$

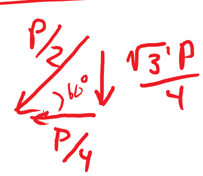
Back to FBD of the machine



$$\Sigma F_y = 2T - P = 0 \Rightarrow \boxed{T = P/2}$$

$$\Sigma F_x = A_x + D_x - \frac{P}{4} = 0$$

$$A_x + D_x = \frac{P}{4}$$



$$\Sigma F_y = A_y + D_y - P - \frac{\sqrt{3}}{4}P = 0$$

$$A_y + D_y = P \cdot \left(\frac{4+\sqrt{3}}{4}\right)$$

$$\Sigma M_A = -(4\,ft)D_x - (4\,ft)\left(\frac{\sqrt{3}}{4}P\right) - (8\,ft) \cdot P = 0$$

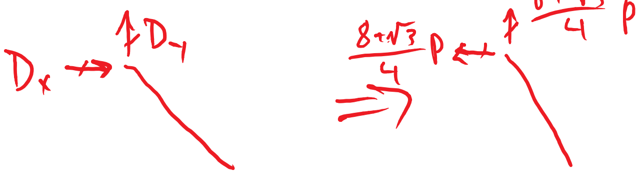
$$\therefore \boxed{D_x = -\left(\frac{8+\sqrt{3}}{4}\right) \cdot P}$$

$$A_x + D_x = \frac{P}{4} \Rightarrow A_x = \frac{P}{4} - D_x = P \cdot \left(\frac{1}{4} + \frac{8}{4} + \frac{\sqrt{3}}{4}\right)$$

$$A_x + D_x = \frac{P}{4} \Rightarrow A_x = \frac{P}{4} - D_x = P\cdot\left(\frac{1}{4} + \frac{8}{4} + \frac{\sqrt{3}}{4}\right)$$

$$\boxed{A_x = (9+\sqrt{3})\cdot\frac{P}{4}}$$

<u>Member BD is a 2-force member</u>, $\theta = 45^\circ \Rightarrow |D_x| = |D_y|$



$$\boxed{D_y = \left(\frac{8+\sqrt{3}}{4}\right)\cdot P} \text{ acts upward}$$

$$\boxed{A_y = P\cdot\left(\frac{4+\sqrt{3}}{4}\right) - D_y = P\cdot\left(\frac{4+\cancel{\sqrt{3}}}{4} - \frac{8}{4} - \cancel{\frac{\sqrt{3}}{4}}\right) = -P}$$

InclassLecture_Chapter6_StructuralAnalysis.pptx

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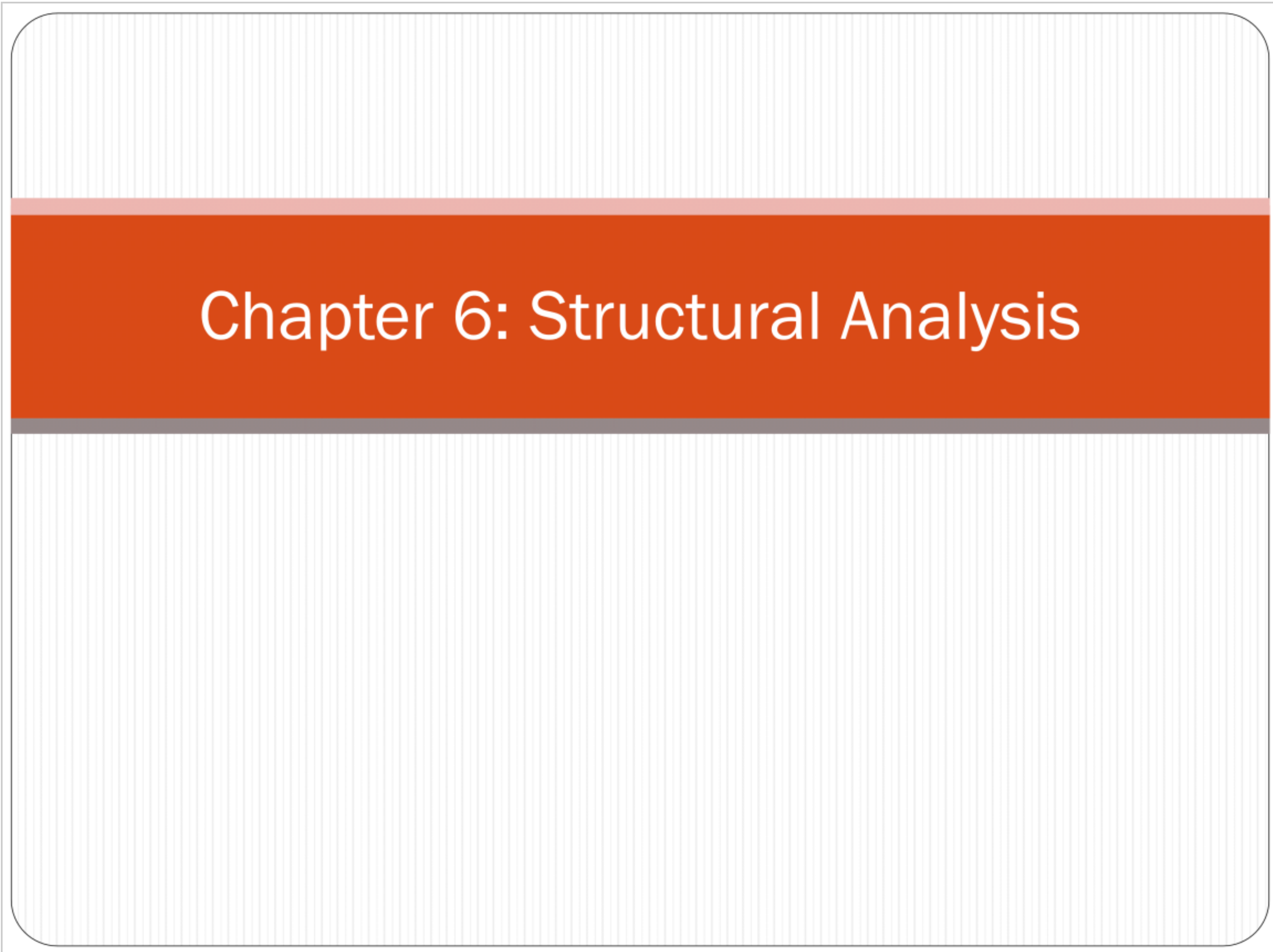

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Scaffolding

An understanding of statics is critical for predicting and analyzing possible modes of failure.

Buckling of slender members in compression is always a consideration in structural analysis.

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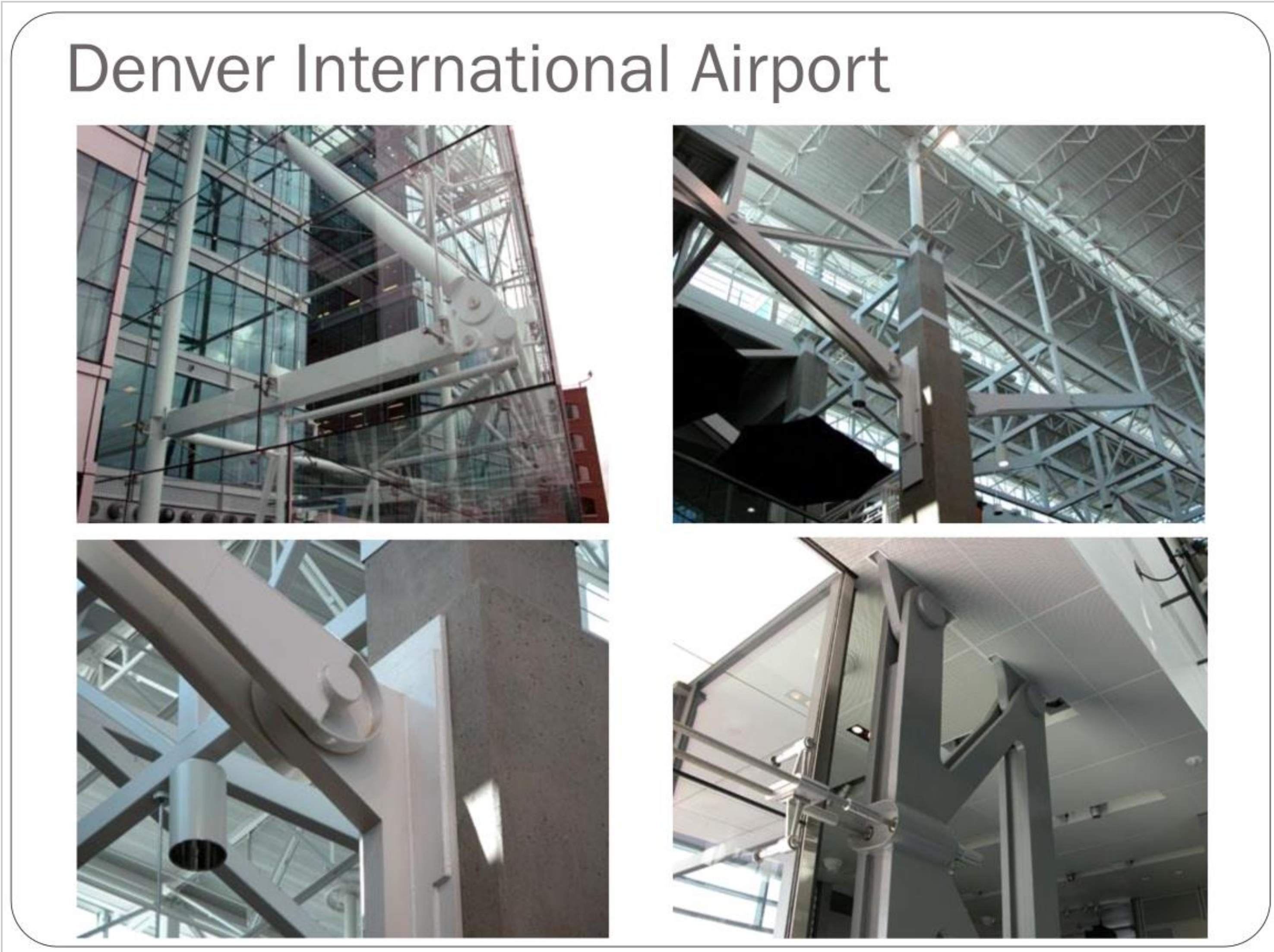

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Simple trusses

Truss:

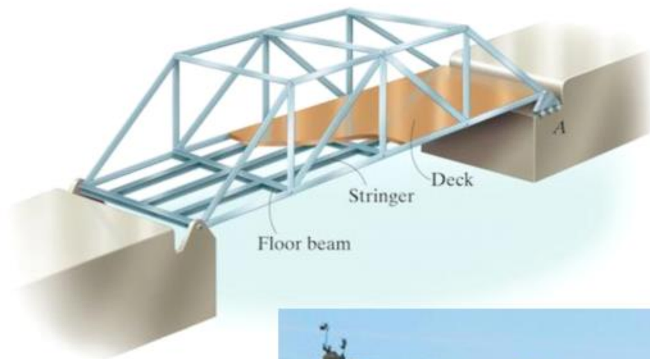
- Structure composed of slender members joined together at end points
- Transmit loads to supports

Assumption of trusses

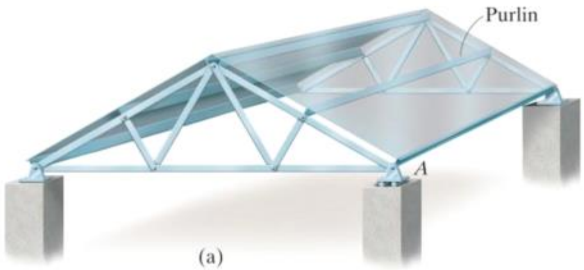
- Loading applied at joints, with negligible weight (If weight included, vertical and split at joints)
- Members joined by smooth pins

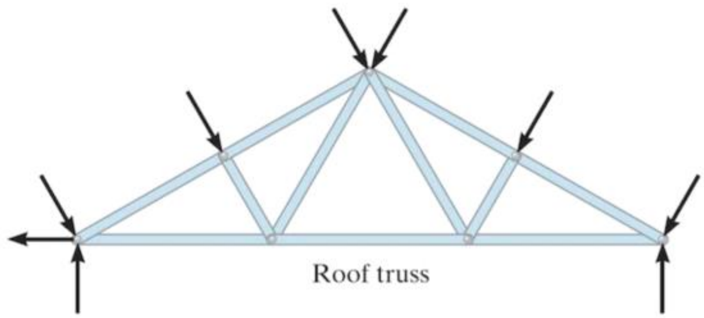
Result: all truss members are two-force members, and therefore the force acting at the end of each member will be directed along the axis of the member

(a)



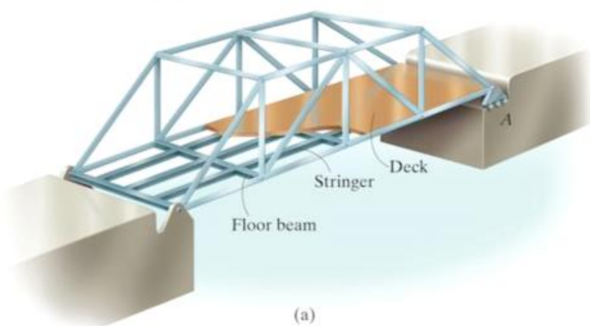
Roof trusses

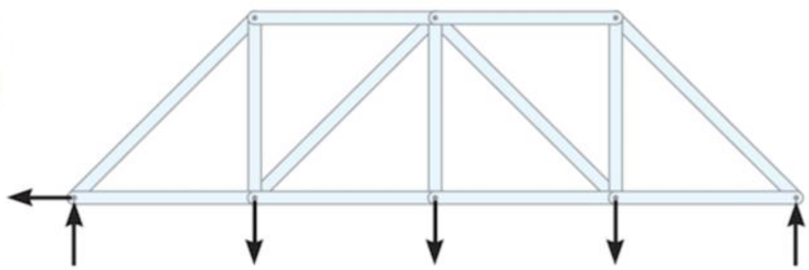




Load on roof transmitted to purlins, and from purlins to roof trusses at joints.

Bridge trusses





Load on deck transmitted to stringers, and from stringers to floor beams, and from floor beams to bridge trusses at joints.

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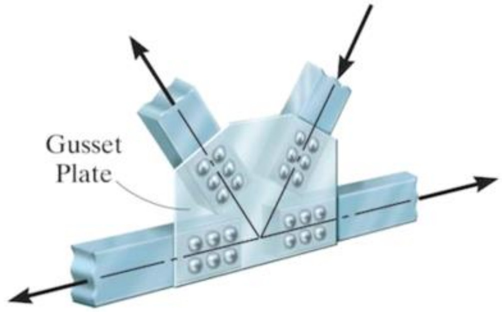
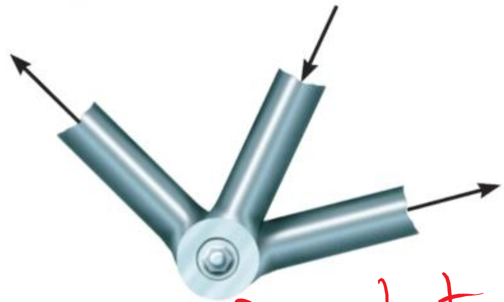
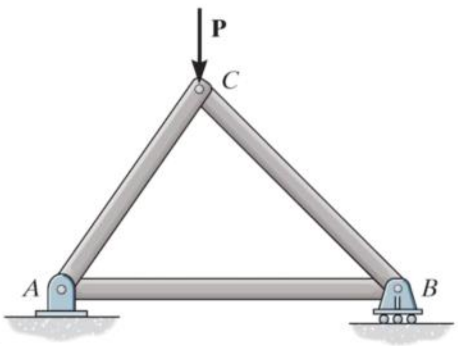
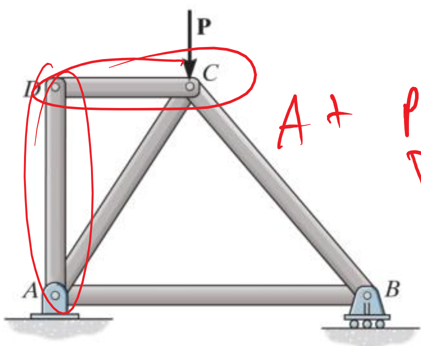
Truss joints

- Bolting or welding of the ends of the members to a gusset plates or passing a large bolt through each of the members
- Properly aligned gusset plates equivalent to pins (i.e., no moments) from coplanar, concurrent forces
- Simple trusses built from triangular members









At pin D, what is D_y?

A) P

B) −P

C) P/2

D) −P/2

E) None

$D_y = 0$

FBD of Pin

$D_x = 0$

$D_y = 0$

Zero-force members